

# Revisiting the Old Theory of Cyclical Growth: Harrod, Kaldor cum Schumpeter

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*This paper advances two main arguments. First, it argues that Harroddian instability can be thought of as the motor force of long period expansions and contractions. This means that the virtuous cycle of an ever-increasing growth rate during the upturn of a long cycle can be seen as a process of runaway expansion, caused by an actual growth rate above the warranted rate. Likewise, the vicious cycle of an ever-deepening downsizing can be interpreted as resulting from an actual growth rate below the warranted rate. Secondly, by showing how a revised Harroddian model can yield a limit cycle in the rate of accumulation, the paper argues that the turning points in these long cycles can be explained by a nonlinear Kaldorian savings function and a variable scrapping rate.*

## 1. Introduction: building on the old debate

The importance of economic downturns for the renewal of capital stock has been a recurrent theme in the history of economic thought. Marx<sup>1</sup> (1967), Veblen (1975, 1964) and Robertson (1915), among others, saw long periods of expansion as cumulative processes that sow the seeds of their own destruction. This occurs because aging capital piles up as deadwood, hindering the adoption of improved techniques of production. According to this argument, economic downturns weed out old and obsolete vintages of capital and set the stage for a renewed expansion. Moreover, as Schumpeter (1934, 1947) emphasized, untapped inventions accumulate during the period of contraction. When a new expansion begins these inventions are put to use, giving rise to a wave of new technology. As the upturn takes hold, imitators jump on the bandwagon. The expansion owes its vibrancy to this wave of diffusion.

Keynes, the intellectual nemesis of this supply-side view, dismissed the claim that economic downturns had a rejuvenating effect. Unlike the modern debate between contemporary supply-siders and the new-Keynesians (which rages over the question of whether rational agents can be influenced as desired

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<sup>1</sup> See vol. I, p. 628; vol. II, p. 170, 186; vol. III, p. 249.

by policy intervention), the old debate focused on whether intervention would lead to industrial stagnation in the long run even if it were effective in the short run.<sup>2</sup> The revision of traditional Harroddian growth theory in this paper owes its inspiration to this old debate and to a stylized reading of the more recent contributions from neo-Schumpeterian economists. It is hoped that the general framework suggested here can help us put in better perspective current discussions about the new economy.

As is well known, Harrod's main insight was that attempts to close capacity shortages (surpluses) by fixed (dis)investment is self-defeating at the aggregate level. A model of long-term cyclical growth can be built on this premise by introducing into the traditional Harroddian analysis a variable rate of savings and scrapping. More specifically, the argument of the paper is based on two premises. First, Harroddian 'instability' can be thought of as the motor force of long-term expansions (contractions).<sup>3</sup> This means that the virtuous cycle of a rising growth rate, caused by an actual growth rate above the warranted rate, can be seen as a process of runaway expansion. In this period, capital shortages are endemic, and efforts to build additional capacity by means of increased investment in fixed capital only exacerbate the general shortage of capital. Likewise, an actual growth rate below the warranted rate gives rise to a vicious cycle of an ever-deepening downsizing, where efforts to get rid off excess capacity by reducing aggregate investment in fixed capital causes only an increase in excess capacity.

Second, the turning points in these long growth cycles can be explained by endogenous changes in the growth rate of potential output in relation to actual output. This paper discusses two mechanisms that can bring about such a result. One is a non-linear savings function *à la* Kaldor. At growth rates of actual output (sufficiently low or high), the savings rate deviates from its *normal* magnitude, giving rise to changes in the growth rate of actual output in relation to that of potential output.

The second mechanism, based on recent neo-Schumpeterian work, involves introducing an endogenous rate of capital obsolescence, which is inversely related to the growth rate of output. During expansions, strong demand checks the diminishing returns to older vintages of capital, lowering the overall rate of scrapping of capital in the economy. Moreover, the pace of obsolescence due technological change is lower, because improvement rather than radical innovations predominate. Likewise, at the bottom of a long-term contraction, the rate of scrapping tends to rise because of weak demand conditions, and because

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<sup>2</sup> For some of these early debates, see Laidler (1999). The rejuvenation effect of the business cycle is a central idea in the Austrian tradition. In Hayek (1939), the sectoral misallocation of bank credit during an expansion distorts the structure of production, which then is 'corrected' during the depression. The thread of the same idea is also found in Minsky (1982), a student of Simons at Chicago and later of Schumpeter at Harvard, who has always insisted that increasing financial fragility during expansions requires as much, if not more, scrutiny as debt-deflation during recessions. See, also, Whalen (1988) and Phillips (1989).

<sup>3</sup> The discussion of the methodological and empirical issues involved in the long wave research remains outside the scope of this paper. For a discussion of the recent work on the long waves, see Kleinknecht *et al.* (1992) and Goldstein (1999).

radical innovation tends to predominate over improvement innovation, speeding up the rate of technological obsolescence associated with older vintages of capital.

The paper specifies the conditions under which these two mechanisms together can endogenously generate turning points in a Harroddian model of runaway expansion and contraction. The rest of the paper is organized into four sections. The next section situates the interpretation of Harrod in this paper in the context of the literature on Harrod. Section 3 reformulates a stylized Harroddian growth model by introducing endogenous variations in the rates of saving and scrapping and discusses its dynamic properties. Section 4 discusses two extensions of the model—the introduction of a variable output–capital ratio and the Verdoorn Law. The last section includes a brief conclusion that summarizes the main argument of the paper.

## **2. Interpretations of Harrod’s Knife Edge**

Harrod’s knife-edge instability has come to have two different meanings, mainly due to the well-known solutions offered by Solow (1956) and Kaldor (1955–6). One meaning concerns deviations of the growth of demand from that of supply. In Harrod’s terminology this deviation is reflected in the discrepancy between the actual and the warranted rates of growth, where the latter is defined as that growth rate of investment that keeps the growth of demand (actual output) and that of supply (potential output) equal to one another.

The second meaning of ‘knife-edge instability’ concerns the discrepancy between the warranted rate and the natural rate. The latter is generally assumed to be equal to some exogenously given growth rate of the labor force and that of labor productivity. This second interpretation ignores possible deviations of the growth of demand from the growth of supply. In other words, actual and potential output cannot differ, and variations in capacity utilization rate are ruled out by assumption. This is the sense in which Solow understands Harroddian instability, and his solution to the problem involves the adjustment of the growth rate of potential output to that of an exogenously given natural rate of growth. In other words, the first meaning of the instability problem is simply ignored. Solow (1994, p. 46) takes this to be one of ‘the defining characteristics of growth theory’ where fluctuations in the capacity utilization rate and similar short run difficulties are ‘papered over’.

The discussion here takes exception to Solow’s methodological approach, and focuses on the first meaning of the knife-edge in the context of long-term growth.<sup>4</sup> This also amounts to assuming an unlimited supply of labor, which is not relaxed until Section 3 below. As is well-known, Solow argued that Harroddian instability is caused by the assumption of fixed production coefficients that are unresponsive to variations in relative factor prices that are sure to come about under conditions of runaway expansion or contraction. However, this argument does not work when actual output is allowed to deviate from potential

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<sup>4</sup> A cursory look at the postwar US data shows that the rate of capacity utilization exhibits short-term fluctuations around a long term cyclical trend.

output. As shown below in Section 3, factor substitution along the lines Solow suggests does not exert a stabilizing influence in the context of the first notion of instability.

In his commentary on Harrod, Kaldor (1955–6) criticizes the assumption of a fixed savings rate and argues that it ought to vary endogenously. The revised Harrodian model below incorporates the idea of an endogenous savings rate, although with conclusions still different from Kaldor's. This is in part due to methodological approach of Kaldor in the 1950s who, similar to Solow, sees Harrodian instability as an argument that pertains to theory of the warranted growth rate, where the actual growth appears only in the context of a mental exercise involving its hypothetical deviation from the warranted growth path (Kregel, 1980). The alternative is to think of it as a theory of actual growth, where the warranted growth path is simply a mental construction against which actual growth can be compared. This has been, by and large, the approach taken by Steindl (1979; 1976, pp. 127–37), who argued that price rigidity under mature capitalism prevents capital stock disequilibrium from 'self-correcting', giving rise to divergent quantity reactions. While in agreement with Steindl's methodological approach, the foregoing does not necessarily corroborate his views on secular stagnation under mature capitalism.

### 3. A Stylized Reformulation of Harrod's Knife Edge

In Domar's (1946) formalization, Harrod's warranted growth rate of investment is the rate that keeps the capacity utilization rate equal to unity—i.e. the rate at which the growth rates of potential and actual output are equal. Assuming that technology is described by constant production coefficients and constant returns to scale, potential output can be expressed as:  $Y_p = \rho K$ , where  $\rho$  is the ratio of potential output to capital stock. Again, making the usual Keynesian assumption that actual output is determined by demand, we can write  $Y = I/s$ , where  $I$  is investment and  $s$  is propensity to save, and assume that there is no depreciation due to the normal wear and tear of capital. This implies that the rate of capacity utilization, here defined as the ratio of actual output to potential output, is given by:

$$u = \frac{Y}{Y_p} = \frac{1}{s\rho} \frac{I}{K} \quad (1)$$

Thus, when  $u = 1$ , the rate of accumulation is equal to the warranted growth rate  $s\rho$ . The accumulation rate responds to any displacement in the rate of capacity utilization, causing the latter to deviate further from unity. This runaway process can be summarized by the following differential equation

$$\dot{z} = \alpha[u(z) - 1] \quad (2)$$

where  $\alpha$  is a coefficient that measures the speed of adjustment,  $z$  is defined as the rate of accumulation ( $I/K$ ) and the dot above it denotes its time rate of change. Figure 1 is a plot of  $u(z)$ , showing the instability implied in Equation (2). At point E, the rate of capacity utilization is equal to unity (and the rate of accumulation is equal to the warranted rate), which implies that  $z = 0$ . To the

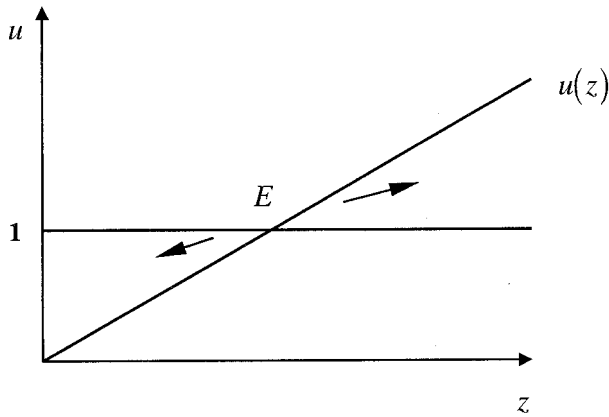


Fig. 1.

right of point  $E$ ,  $z > 0$ , which means that the rate of accumulation is rising when  $u > 1$ . To the left of  $E$ ,  $u < 1$  and thus  $z < 0$ .

We now make two modifications to this stylized Harrodian equation. The first involves incorporating a variable saving rate *à la* Kaldor.<sup>5</sup> At sufficiently low and high rates of accumulation, the propensity to save is lower and larger, respectively, than its *normal* value in the intermediate range of  $z$  (see Fig. 2). This might be so, as Kaldor argues, because of bottlenecks caused by sectors with inelastic supplies (and thus rising rents) when  $z$  is very high, and of

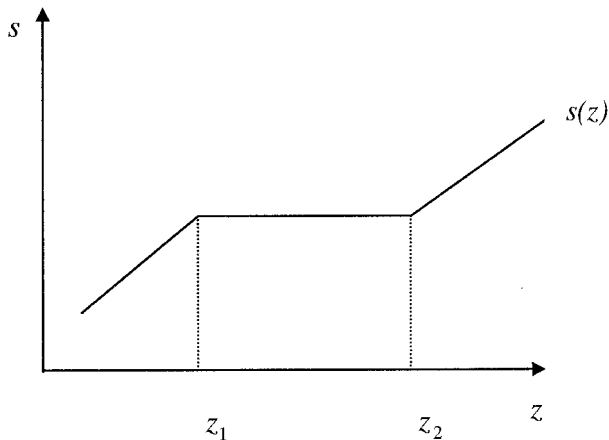


Fig. 2.

<sup>5</sup> The idea of a variable saving rate first appears in Kaldor's (1940) trade cycle theory in the context of a nonlinear saving behavior. The discussion below is closer to the treatment in his trade cycle analysis than in his later work on growth theory.

consumers' efforts to maintain consumption at accustomed levels by running down accumulated wealth when  $z$  starts to get very low.<sup>6</sup>

This implies that the capacity utilization rate is a function of both the rate of accumulation and the propensity to save:  $u = u[s(z), z]$ , where  $u_s < 0$  and  $u_z > 0$  from Equation (1). The slope of the capacity utilization function is given by:

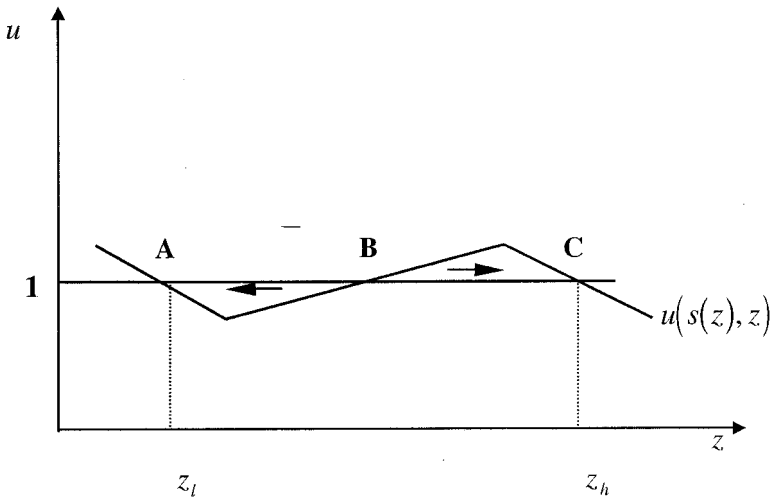
$$\frac{du}{dz} = u_s \frac{ds}{dz} + u_z.$$

The nonlinearity in the saving function implies that  $ds/dz$  is positive when  $z$  takes sufficiently high or low values and is equal to zero for the intermediate values of  $z$ . In other words, a lower bound,  $z < z_1$ , and a higher bound,  $z > z_2$ , exists where  $ds/dz > 0$ , and  $ds/dz = 0$  when  $z_1 < z < z_2$ . In this formulation, two conditions have to be met for a variable saving rate to contain the instability problem. For all values of  $z < z_1$  and  $z > z_2$ , it must be the case that

$$|u_s| \frac{ds}{dz} > u_z,$$

and, within this range, the savings rate must be a monotonically increasing function of the rate of accumulation,  $z$ , as depicted in Fig. 2. Provided that these conditions are met the capacity utilization function intersects unity thrice, as depicted in Fig. 3.

The second modification to Equation (2) involves introducing capital scrapping as a shift variable in the capacity utilization function. It is assumed



**Fig. 3.**

<sup>6</sup> Thus, the savings rate here should be interpreted not as the behavioral propensity based on household psychology but the macroeconomic rate that includes forced savings as well.

that the rate of scrapping is a slowly changing negative function of the growth rate of actual output. This assumption is based on two considerations. First, strong (low) demand associated with a high (low) growth rate of output is thought to reduce (raise) the rate of scrapping as it raises the quasi rents associated with the older vintages of capital. Second, a stylized reading of recent neo-Schumpeterian work suggests that the propensity of technological change to cause obsolescence varies negatively with the growth rate of output. While some innovations wipe out the rents associated with the older vintages of capital, others—which are complementary with the old technology—might have little effect on, or even enhance, these rents. Here, the former are referred to as radical and the latter as improvement innovations, and it is assumed that only radical innovations cause premature obsolescence and thus cause changes in the rate of scrapping. Section 4 discusses the implications of relaxing this assumption and of the possible positive impact the growth rate output might have on technological change (the so-called Verdoorn Law).

In the neo-Schumpeterian literature, radical innovations are described as those giving rise to a family of new products, needs, industries and markets. These new products mature by means of a series of quality-augmenting and cost-reducing improvement innovations (Freeman, 1984, 1989). As long as markets keep expanding firms might stick to quality improvements and cost reductions within existing industries and technologies. This implies that the very success of efforts in improving existing technologies can impede the development of completely new technologies—the so-called sailing ship effect (Rosenberg, 1982). As a result, few radical innovations are introduced during periods of robust expansion, and technological change progresses within the same paradigm without causing much destruction to the rents of capital already in operation.<sup>7</sup> By contrast, during prolonged periods of stagnation and low growth, firms might be forced to shift their attention to efforts to commercialize untapped inventions that have been accumulating in the preceding period of expansion. As Nelson & Winter (1982) remark, adversity stimulates firms to depart from the normal way of doing things and to search for radical innovations.<sup>8</sup> The cumulative effect of this reorientation is an increase in radical innovations that are detrimental to the rents of older vintages of capital.

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<sup>7</sup> According to Dosi (1982), the progress of technological innovations can be thought of along lines very similar to the distinction Kuhn has drawn between normal science which progresses within a given paradigm, and those scientific advances that give rise to a new paradigm. In a similar vein, Freeman & Perez (1988) talk about a family or a life-cycle of innovations, while Nelson & Winter (1982) coin the term natural trajectories to convey what is basically the same idea.

<sup>8</sup> A stronger version of this argument originates from Mensch (1979) who maintained that radical innovations are triggered by depressions. Although his empirical findings have been much criticized (Clark *et al.* 1981), there has emerged over time a qualified support for the broad contours of his argument, if not for its details. First, it is generally agreed that radical innovations occur in waves if not in tight clusters as Mensch has originally argued (Kleinknecht 1987, 1990). Bunching of innovations takes place more in the early phases of a recovery following a depression, indicating that depressions have a temporary retarding effect on radical innovations. Secondly, radical innovations increasingly give way to improvement and rationalizing innovations as expansions progress (van Duijn, 1981, 1983; Graham & Senge 1980, Freeman *et al.* (1982). For a skeptic's view, see also Solomou (1986).

Defining the rate of scrapping as the ratio of the magnitude of capital discarded,  $K^d$ , in the unit period to the capital stock,

$$\varepsilon = \frac{K^d}{K},$$

potential output can now be expressed as  $Y_p = \rho(1 - \varepsilon)K$ , and Equation (1) is transformed into;

$$u = \frac{z}{s\rho(1 - \varepsilon)} \quad (3)$$

These two modifications, in turn, yield the following set of equations:

$$\begin{aligned} z &= \alpha[u(s(z), z, \varepsilon) - 1] \\ \dot{\varepsilon} &= \varphi(z) \end{aligned} \quad (4)$$

where, in the first equation, the rate of scrapping,  $\varepsilon$ , appears as a shift variable, and from Equation (3) it follows that  $u_\varepsilon > 0$ . In the second equation, the rate of change of the scrapping rate,  $\dot{\varepsilon}$ , is expressed as an inverse function of the rate of accumulation,  $\varphi'(z) < 0$ . It is assumed that there is some unique value of rate of accumulation,  $z^*$ , within the range,  $z_1 < z < z_2$ , for which  $\varphi(z^*) = 0$ , i.e. the rate of scrapping is equal to its *normal* value,  $\varepsilon = \bar{\varepsilon}$ , such that  $\dot{\varepsilon} = 0$ . As we shall see, when  $z = z^*$ , the condition  $u(s(z), z, \bar{\varepsilon}) = 1$ , i.e.  $z = 0$ , also holds. Both functions are assumed to be continuously differentiable.

As shown in Fig. 3, three different values of the rate of accumulation yield a unitary rate of capacity utilization. The position of equilibrium B in the middle is unstable, while the other two on the sides A and C are stable. During an expansion the economy moves toward the high equilibrium position at point C, and in the meantime the higher rate of accumulation leads to a fall in the rate of scrapping which, as shown below, gives rise to endogenous cycles in the rate of accumulation. The fall in the rate of scrapping in turn shifts the capacity utilization function downward, closing the gap between points B and C until the two points merge and eventually disconnect, causing a cumulative fall in the rate of accumulation toward point A (Fig. 4). During contraction the same process works in reverse: the rise in the rate of scrapping pushes up the capacity utilization function until points A and B merge and then disconnect, leaving C (which has been moving to the right) as the only equilibrium point towards which the economy gravitates.

The dynamic properties of the equation system in (4) can be discussed by means of a graphical argument. The slope of the  $z = 0$  isocline in Fig. 5 is given by

$$\left. \frac{d\varepsilon}{dz} \right|_{z=0} = \frac{-(u_s s_z + u_z)}{u_\varepsilon}$$

The denominator is always positive and, since the sign of  $(u_s s_z + u_z)$  is negative for  $z < z_1$  and  $z > z_2$ , and positive for  $z_1 < z < z_2$ , the sign of the  $z = 0$  isocline must be positive for  $z < z_1$  and  $z > z_2$ , and negative in the range  $z_1 < z < z_2$ . Because the rate of scrapping is not influenced by the savings rate, ( $\varphi_s = 0$ ), the



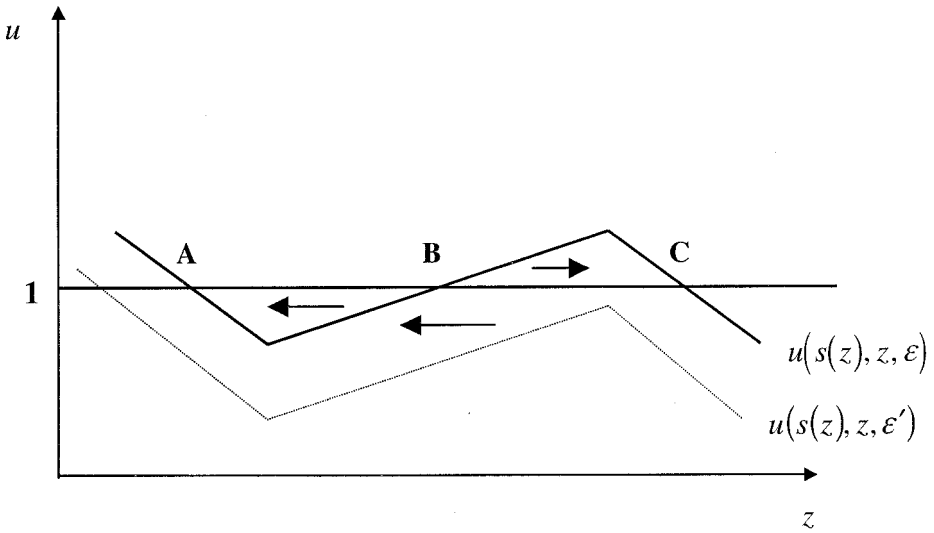


Fig. 4.

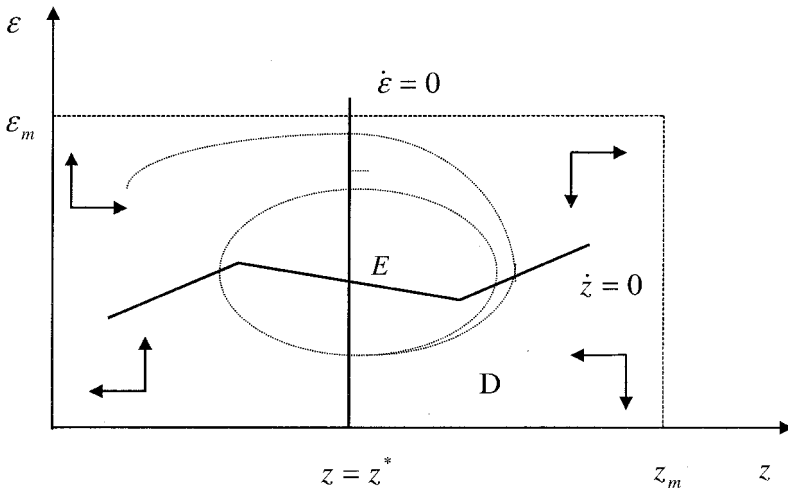


Fig. 5.

$\dot{\varepsilon} = 0$  isocline is the vertical line  $z = z^*$ . The point at which the two isoclines intersect is the fixed point at which both variables are stationary, ( $\dot{z} = 0$  and  $\dot{\varepsilon} = 0$ ). Thus, when  $z = z^*$  the condition  $u(s(z, z, \bar{\varepsilon})) = 1$ , i.e.  $\dot{z} = 0$ , also holds.

Evaluating the Jacobian matrix of the linearized version of the equation system around this fixed point;

$$J_E = \begin{bmatrix} \alpha(u_s s_z + u_z) & \alpha u_\varepsilon \\ \varphi_z & 0 \end{bmatrix}$$

it can be seen that the sum of the two characteristic roots is given by the

$\text{tr}J_E = \alpha(u_s s_z + u_z)$ , and their product by  $|J_E| = -\alpha u_\varepsilon \varphi_z$ . Since  $\varphi_z < 0$ ,  $u_\varepsilon > 0$ ,  $u_z > 0$  throughout, and  $s_z = 0$  for  $z_1 < z < z_2$ , the determinant and the trace of the Jacobian matrix must both be positive, which means that the singular point is unstable. In the range,  $z < z_1$  and  $z < z_2$ , the trace changes sign and becomes negative since  $s_z > 0$ ,  $s(z)$  is monotonically increasing and  $|u_s|s_z > u_z$ , implying that the equation system in (4) exhibits a limit cycle.

If we denote the compact set D of R as the set  $\{(z, \varepsilon) | 0 \leq z \leq z_m, 0 \leq \varepsilon < \varepsilon_m\}$  then any positive semi-orbit in R starting outside D will eventually enter D and, once in, its trajectory cannot exit D. That the system exhibits limit cycles around this fixed point can be shown by noting that the vector field points inwards. Since  $\partial z / \partial \varepsilon = u_\varepsilon > 0$ , at any point that lies above (below) the  $\dot{z} = 0$  isocline,  $dz/dt$  must be positive (negative). Likewise, since  $\partial \varepsilon / \partial z = \varphi_z < 0$ , at any point to the left (right) of the  $\dot{\varepsilon} = 0$  isocline  $d\varepsilon/dt$  is positive (negative). Since the singularity of the linearized system is shown to be unstable, and in the bounded region D no singular point other than E exists, and it is impossible for a trajectory within the region to exit, then by the theorem of Poincaré-Bendixson there exists in D at least one attracting closed orbit.

#### 4. Extensions

If we drop the functional relationship between scrapping and the rate of accumulation, the model here mimics Kaldor's discussion of Harrod, except here the system does not gravitate around an exogenously given and unique natural growth rate as in Kaldor's discussion. Instead, points A and C in Fig. 3 emerge as stable positions where a unitary capacity utilization rate holds at two different rates of accumulation corresponding to these points, while point B still remains unstable. The outcome at point C is similar to what Keynes once termed a 'permanent quasi-boom' while point A is a long lasting slump. However, the 'permanent' boom cannot be permanent if the rate of scrapping begins to fall during expansion. The rate of accumulation gradually diminishes over time as the capacity utilization function begins to shift down, leading eventually to a cumulative process of downsizing that only stabilizes in a slump, which might resemble Steindl's 'secular stagnation'.

Next, we consider the implications of (i) relaxing our assumption of a constant potential output–capital ratio, and (ii) introducing Verdoorn (or Kaldor's) Law.

##### (i) *Factor Substitution*

Although the Cobb–Douglas aggregate production function that Solow used is not compatible with the methodological orientation<sup>9</sup> of the model here, the idea that long-term shifts in relative factor costs can influence the way production coefficients evolve over time can be introduced without any difficulty (Nelson,

<sup>9</sup> On different grounds, both post-Keynesian and neo-Schumpeterian economists have argued that shifts in the aggregate Cobb–Douglas production function cannot be distinguished from movements along it. See also Joan Robinson's (1953–4) well-known critique that set off the capital controversy.

1994). If rising real wages contribute to the adoption of more capital intensive technologies, and if increases in capital intensity give rise to falling output per unit capital as implied in the Cobb–Douglas production function, the intuitive idea behind factor substitution can be introduced by positing an inverse relationship between the output capital ratio and the rate of accumulation.<sup>10</sup> A higher rate of accumulation would imply a tighter labor market and thus higher real wages; and, that, in turn, would induce a rise in capital intensity, implying a fall in the output–capital ratio. Thus, re-writing Equation (1) as,

$$u = \frac{z}{s(z)\rho(z)(1 - \varepsilon)} \quad (5)$$

where  $\rho'(z) < 0$ , the first equation of the equation system in (4) becomes:

$$\dot{z} = \alpha[u(s(z), \rho(z), z, \varepsilon) - 1] \quad (6)$$

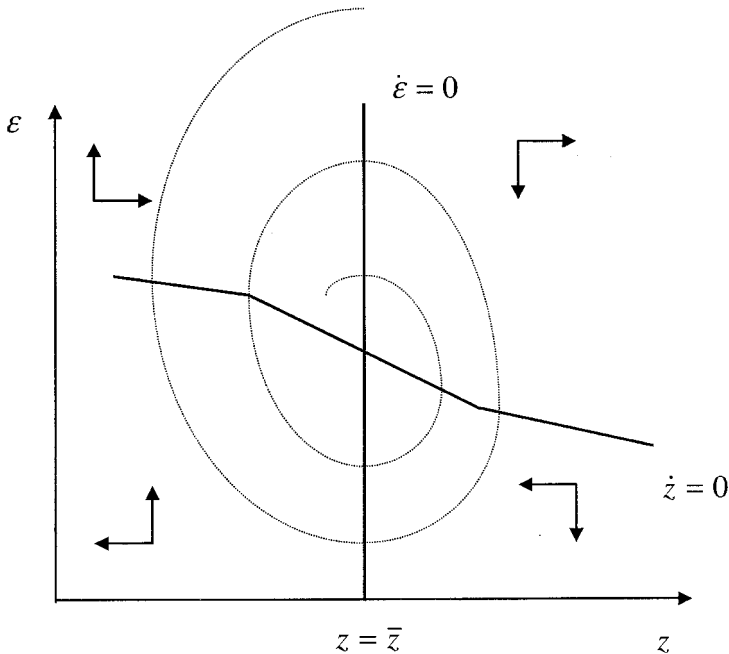
The Jacobian of the revised system now yields:  $\text{tr}J_E = u_s s_z + u_\rho \rho_z + u_z$  while  $|J_E|$  is unaltered. Since both  $u_\rho$  and  $\rho_z$  are negative, the condition for the system to exhibit a limit cycle becomes more stringent. The capacity utilization rate rises not only with the rate of accumulation but also with the falling potential output capital ratio. Thus, the nonlinearity in the saving function has to be strong enough to counteract the positive impact of both on capacity utilization. Either the low and high rates of accumulation for which  $|u_s s_z| > |u_\rho \rho_z + u_z|$  become respectively lower and higher (in which case both the expansions and contractions are prolonged) or  $u_s s_z$  remains in absolute value lower than  $u_\rho \rho_z + u_z$  for all possible values of the rate of accumulation, in which case the system turns into an unstable focus (see Fig. 6). Thus, contrary to Solow’s conclusion, once the capacity utilization rate is allowed to vary, factor substitution along the lines he suggested is either not *stabilizing* or *destabilizing*.<sup>11</sup>

### (ii) Verdoorn Law

The implications of the Verdoorn Law and the broader hypothesis that innovations are demand-pulled (Schmookler, 1966), for the foregoing discussion, depend on whether these arguments apply to improvement innovations only or to radical innovations as well. If only improvement innovations are positively related to the growth rate of output, while radical innovations ‘follow a “counter-Schmookler” pattern’ as Brouwer & Kleinknecht (1999) suggest and other neo-Schumpeterians argue, then the Verdoorn Law can be introduced into the discussion by revising only the first equation in (4). Positing a positive relationship between the output–capital ratio and the growth rate of output, we

<sup>10</sup> The argument that its productivity would diminish as capital is used more intensively in production, although controversial, is not uncommon among heterodox and, especially, Marxian economists. Also, an affinity exists between the Kaldor & Mirrlees (1962) technological progress function and the neo-Schumpeterian ‘natural trajectories of technological change.’ In both, production coefficients evolve over time with changes in technology, and returns to technical change increase at a diminishing rate as the rate of accumulation increases.

<sup>11</sup> I thank Tracy Mott for alerting me to the fact that Dobb (1973, pp. 230–231) makes a similar argument.



**Fig. 6.**

can again re-write Equation (1) as in (5) and revise the first equation in (4) as in (6). But now  $\rho_z$  would be positive, and the impact of the Verdoorn Law would be exactly the opposite of what has been remarked with respect to factor substitution as discussed above in (i). The rate of capacity utilization still rises with accumulation but now falls with a rising potential output capital ratio, suggesting that the condition for cyclical behavior becomes less stringent and that expansions and contractions are shortened. This implies that the sign of  $\rho_z$  would depend on which of the two effects (factor substitution and Verdoorn Law) is stronger, while it is also possible that they can cancel each other out.

The situation is rather different if the Verdoorn Law also applies to radical innovations or if improvement innovations give rise to as much scrapping as do radical innovations. In this case, the dynamics of the model revert back to a knife-edge similar to that of Harrod.<sup>12</sup> To the unstable interaction of capacity utilization with the rate of accumulation, is now added that with technological innovation. A higher rate of accumulation gives rise to a faster pace of technological change (of whichever type), increasing the rate of scrapping; while, that, in turn, increases the capacity utilization rate, stimulating further the pace of accumulation and technological innovation.

<sup>12</sup> In the second equation in (4),  $Q_z$  becomes negative and thus  $|J_E| < 0$ , implying a saddle point solution.

## 5. Conclusions: main argument restated

The paper has argued that the principle of Harroddian instability can be used in explaining long-period expansions and contractions. This means that virtuous cycles of ever-rising growth rates during an upturn can be thought of as a runaway process caused by an actual growth rate above the warranted rate. Likewise, vicious cycles of an ever-deepening process of downsizing can be interpreted to result from an actual growth rate that lies below the warranted rate. The turning points, in turn, can be explained by a variable saving rate and an endogenous rate of scrapping, provided that the savings rate reacts sufficiently strongly in the face of a runaway expansion or contraction, as Kaldor maintained it would.

## References

- Brouwer, E. & Kleinknecht, A. (1999) Keynes-plus? Effective demand and changes in firm-level R&D: an empirical note, *Cambridge Journal of Economics*, 23(3), pp. 385–91.
- Clark, J., Freeman, C. & Soete, L. (1981) Long waves and inventions, and innovations, *Futures*, 13, pp. 308–322.
- Dobb, M. (1973) *Theories of Value and Distribution since Adam Smith* (Cambridge, Cambridge University Press).
- Domar, E. (1946) Capital expansion, rate of growth and employment, *Econometrica*, 14, pp. 137–147.
- Dosi, G. (1982) Technological paradigms and technological trajectories, *Research Policy*, 11, pp. 147–164.
- Dosi, G. *et al.* (1988) *Technical Change and Economic Theory* (New York, Pinter).
- Freeman, C. (1984) Prometheus unbound, *Futures*, 16, pp. 494–507.
- Freeman, C. (1989) Technical change and depression in the 1930s and 1980s, in: M. Di Matteo, R. M. Goodwin, A. Vercelli (Eds.) *Technological and Social Factors in Long Term Fluctuations* (New York, Springer-Verlag), pp. 419–42.
- Freeman, C. & Perez, C. (1988) Structural crises of adjustment, business cycles and investment behaviour, in: G. Dosi *et al.* (Ed.) *Technical Change and Economic Theory* (New York, Pinter), pp. 38–66.
- Freeman, C., Clark, C. & Soete, L. (1982) *Unemployment and Technical Change* (London, Francis Pinter).
- Graham, A. K. & Senge, P. M. (1980) A long-wave hypothesis on innovation, *Technological Forecasting and Social Change*, 17, pp. 283–296.
- Goldstein, J. P. (1999) The existence, endogeneity, and synchronization of long waves, *Review of Radical Political Economics*, 31(4), pp. 61–101.
- Harrod, R. (1939) An essay in dynamic theory, *Economic Journal*, 49, pp. 14–33.
- Hayek, F. A. (1939) *Profit, Interest and Investment* (London, Routledge & Kegan Paul).
- Kaldor, N. (1940) A model of the trade cycle, *Economic Journal*, 50, pp. 78–92.
- Kaldor, N. (1955–6) Alternative theories of distribution, *Review of Economic Studies*, 23, pp. 94–100.
- Kaldor, N. & Mirrlees, J. (1962) A new model of economic growth, *The Review of Economic Studies*, 29(3), pp. 174–192.
- Kleinknecht, A., (1987) *Innovation Patterns in Crisis and Prosperity: Schumpeter's long cycle reconsidered* (New York, St. Martin's Press).
- Kleinknecht, A. (1990) Are there Schumpeterian waves of innovation?, *Cambridge Journal of Economics*, 14, pp. 81–92.
- Kleinknecht, A. Mandel, E. & Wallerstein, I. (1992) *New Findings in Long-wave Research* (New York, St. Martin's Press).
- Kregel, J. (1980) Economic dynamics and the theory of steady growth, *History of Political Economy*, 12, pp. 97–123.

- Laidler, D. (1999) *Fabricating the Keynesian Revolution* (New York, Cambridge University Press).
- Marx, K. (1967, [1867]) *Capital*, Vol. I (New York, International Publishers).
- Marx, K. (1967, [1893]) *Capital*, Vol. II (New York, International Publishers).
- Marx, K. (1967, [1894]) *Capital*, Vol. III (New York, International Publishers).
- Mensch, G. (1979) *Stalemate in Technology* (Cambridge, Ballinger).
- Minsky, H. (1982) *Can it Happen Again? Essays on instability and finance* (Armonk, NY, M.E. Sharpe).
- Nelson, R., (1994) What has been the matter with neoclassical growth theory? in: G. Silverberg & L. Soete (Eds.) *The Economics of Growth and Technological Change* (Brookfield, VT, Edward Elgar), pp. 290–324.
- Nelson, R. & Winter, S. (1982) *An Evolutionary Theory of Technical Change* (Cambridge, Harvard University Press).
- Phillips, R. J. (1989) The Minsky-Simons-Veblen connection: comment, *Journal of Economic Issues*, 23(3), pp. 889–891.
- Robertson, D. H. (1915) *A Study of Industrial Fluctuation* (London, P. S. King & Son).
- Robinson, J. (1953–4) The production function and the theory of capital, *Review of Economic Studies*, 21, pp. 81–106.
- Rosenberg, N. (1982) *Inside the Black Box: technology and economics* (Cambridge, Cambridge University Press).
- Schmookler, J. (1966) *Invention and Economic Growth* (Cambridge, MA, Harvard University Press).
- Schumpeter, J. (1934) *The Theory of Economic Development: an inquiry into profits, capital, credit, interest, and the business cycle* (Cambridge, Harvard University Press).
- Schumpeter, J. (1947) *Capitalism, Socialism and Democracy*, 2nd edn (New York, Harper & Brothers).
- Solomou, S. (1986) Innovation clusters and Kondratieff long waves in economic growth, *Cambridge Journal of Economics*, 10, pp. 101–112.
- Solow, R. (1956) A contribution to the theory of economic growth, *Quarterly Journal of Economics*, 70, pp. 65–94.
- Solow, R. (1994) Perspectives on growth theory, *The Journal of Economic Perspectives*, 8, pp. 45–54.
- Steindl, J. (1976) *Maturity and Stagnation in American Capitalism* (New York, Monthly Review Press).
- Steindl, J. (1979) Stagnation theory and stagnation policy, *Cambridge Journal of Economics*, 3, pp. 1–14.
- van Dujin, J. J. (1981) Fluctuations of innovations over time, *Futures*, 13, pp. 264–275.
- van Dujin, J. J. (1983) *The Long Wave in Economic Life* (London, Allen & Unwin).
- Veblen, T. (1964, [1923]) *Absentee Ownership and Business Enterprise in Recent Times; the case of America*. (New York, A.M. Kelley).
- Veblen, T. (1975, [1904]) *The Theory of Business Enterprise* (Clifton, NJ, A.M. Kelley).
- Whalen, C. (1988) The Minsky-Simons connection: a neglected thread in the history of economic thought, *Journal of Economic Issues*, 22, pp. 533–544.